

# CHAPTER 2

## Fluid Statics

### FE-type Exam Review Problems: Problems 2-1 to 2-9

2.1 (C)  $p = \gamma_{Hg} h = (13.6 \times 9810) \times (28.5 \times 0.0254) = 96\,600 \text{ Pa} = 96.6 \text{ kPa}$

2.2 (D)  $p = p_0 - \rho g h = 84\,000 - 1.00 \times 9.81 \times 4000 = 44\,760 \text{ Pa}$

2.3 (C)  $p_w = p_{atm} + \gamma_x h_x - \gamma_{water} h_w = 0 + 30\,000 \times 0.3 - 9810 \times 0.1 = 8020 \text{ Pa} = \underline{8 \text{ kPa}}$

2.4 (A)  $p_a = -\gamma H = -(13.6 \times 9810) \times 0.16 = -21\,350 \text{ Pa}$

$$p_{a,after} = -21\,350 + 10\,000 = -11\,350 = 13.6 \times 9810 H_{after} \therefore H_{after} = 0.0851 \text{ m}$$

2.5 (B) The force acts 1/3 the distance from the hinge to the water line:

$$\left(2 \times \frac{5}{3}\right) P = \frac{1}{3} \left(2 \times \frac{5}{3}\right) \left[9810 \times 1 \times 3 \times \left(2 \times \frac{5}{3}\right)\right]$$

$$\therefore P = 32\,667 \text{ N} = \boxed{32.7 \text{ kN}}$$

2.6 (A) The gate opens when the center of pressure is at the hinge:

$$\bar{y} = \frac{1.2+h}{2} + 5. \quad y_p = \bar{y} + \frac{\bar{I}}{A\bar{y}} = \frac{11.2+h}{2} + \frac{b(1.2+h)^3/12}{(1.2+h)b(11.2+h)/2} = 5 + 1.2.$$

This can be solved by trial-and-error, or we can simply substitute one of the answers into the equation and check to see if it is correct. This yields  $h = 1.08 \text{ m}$ .

2.7 (D) Place the force  $\bar{F}_H + \bar{F}_V$  at the center of the circular arc.  $F_H$  passes through the hinge:

$$\therefore P = F_V = 4 \times 1.2w \times 9800 + (\pi \times 1.2^2/4)w \times 9800 = 300\,000. \quad \therefore w = 5.16 \text{ m}.$$

2.8 (A)  $W = \gamma V$

$$900 \times 9.81 = 9810 \times 0.01 \times 15w. \quad \therefore w = 6 \text{ m}$$

2.9 (A)  $p_{plug} = 20\,000 + \gamma h = 20\,000 + 6660 \times \left(1.2 \times \frac{5}{9.81}\right) = 24\,070 \text{ Pa}$

$$F_{plug} = p_{plug} A = 24\,070 \times \pi \times 0.02^2 = 30.25 \text{ N}.$$

Chapter 2 Problems: Pressure

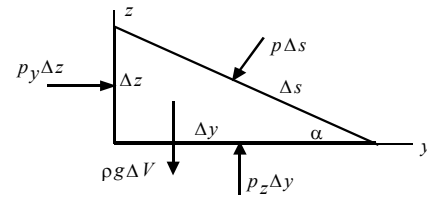
$$2.10 \quad \Sigma F_y = ma_y: \quad p_y \Delta z - p \Delta s \sin \alpha = \rho \frac{\Delta y \Delta z}{2} a_y$$

$$\Sigma F_z = ma_z: \quad p_z \Delta y - p \Delta s \cos \alpha = \rho \frac{\Delta y \Delta z}{2} a_z + \rho g \frac{\Delta y \Delta z}{2}$$

Since  $\Delta s \cos \alpha = \Delta y$  and  $\Delta s \sin \alpha = \Delta z$ , we have

$$p_y - p = \rho \frac{\Delta y}{2} a_y \quad \text{and} \quad p_z - p = \rho \frac{\Delta z}{2} (a_z + g)$$

$$\text{Let } \Delta y \rightarrow 0 \text{ and } \Delta z \rightarrow 0: \quad \text{then } \left. \begin{array}{l} p_y - p = 0 \\ p_z - p = 0 \end{array} \right\} \quad \therefore p_y = p_z = p.$$



- 2.11  $p = \gamma h$ .
- a)  $9810 \times 10 = 98\,100 \text{ Pa}$  or 98.1 kPa
  - b)  $(0.8 \times 9810) \times 10 = 78\,480 \text{ Pa}$  or 78.5 kPa
  - c)  $(13.6 \times 9810) \times 10 = 1\,334\,000 \text{ Pa}$  or 1334 kPa
  - d)  $(1.59 \times 9810) \times 10 = 155\,980 \text{ Pa}$  or 156.0 kPa
  - e)  $(0.68 \times 9810) \times 10 = 66\,710 \text{ Pa}$  or 66.7 kPa

- 2.12  $h = p/\gamma$ .
- a)  $h = 250\,000/9810 = \underline{25.5 \text{ m}}$
  - b)  $h = 250\,000/(0.8 \times 9810) = \underline{31.9 \text{ m}}$
  - c)  $h = 250\,000/(13.6 \times 9810) = \underline{1.874 \text{ m}}$
  - d)  $h = 250\,000/(1.59 \times 9810) = \underline{16.0 \text{ m}}$
  - e)  $h = 250\,000/(0.68 \times 9810) = \underline{37.5 \text{ m}}$

2.13  $S = \frac{p}{\gamma h} = \frac{138 \times 10^3}{10^3 \times 9.81 \times 61} = 0.231 \quad \rho = 0.231 \times 10^3 = \boxed{231 \frac{\text{kg}}{\text{m}^3}}$

- 2.14
- a)  $p = \gamma h = 0.76 \times (13.6 \times 9810) = 9810 h. \quad \therefore h = \underline{10.34 \text{ m}}$ .
  - b)  $(13.6 \times 9810) \times 0.75 = 9810 h. \quad \therefore h = \underline{10.2 \text{ m}}$ .
  - c)  $(13.6 \times 9810) \times 0.01 = 9810 h. \quad \therefore h = 0.136 \text{ m}$  or 13.6 cm.

- 2.15
- a)  $p = \gamma h_1 + \gamma h_2 = 9810 \times 0.2 + (13.6 \times 9810) \times 0.02 = 4630 \text{ Pa}$  or 4.63 kPa.
  - b)  $9810 \times 0.052 + 15\,630 \times 0.026 = 916 \text{ Pa}$  or 0.916 kPa.
  - c)  $9016 \times 3 + 9810 \times 2 + (13.6 \times 9810) \times 0.1 = 60\,010 \text{ Pa}$  or 60.0 kPa.

2.16  $\Delta p = \rho g \Delta h = 1.27 \times 9.81 \times (-3000) = -37370 \text{ Pa} = -37.4 \text{ kPa}$ .

$$2.17 \quad \left. \begin{aligned} \Delta p_{outside} &= \rho_o g \Delta h = \frac{pg}{RT_o} \Delta h = \frac{100 \times 9.81}{0.287 \times 253} \times 3 = 13.51 \text{ Pa} \\ \Delta p_{inside} &= \rho_i g \Delta h = \frac{pg}{RT_i} \Delta h = \frac{100 \times 9.81}{0.287 \times 293} \times 3 = 11.67 \text{ Pa} \end{aligned} \right\} \therefore \Delta p_{base} = \underline{1.84 \text{ Pa}}$$

If no wind is present this  $\Delta p_{base}$  would produce a small infiltration since the higher pressure outside would force outside air into the bottom region (through cracks).

$$2.18 \quad p = \rho g dh \text{ where } h = -z. \text{ From the given information } S = 1.0 + h/100 \text{ since } S(0) = 1 \text{ and } S(10) = 1.1. \text{ By definition } \rho = 1000 S, \text{ where } \rho_{water} = 1000 \text{ kg/m}^3. \text{ Then } dp = 1000 (1 + h/100) g dh. \text{ Integrate:}$$

$$\int_0^p dp = \int_0^{10} 1000(1 + h/100) g dh$$

$$p = 1000 \times 9.81 \left( 10 + \frac{10^2}{2 \times 100} \right) = 103 \text{ 000 Pa or } \underline{103 \text{ kPa}}$$

Note: we could have used an average  $S$ :  $S_{avg} = 1.05$ , so that  $\rho_{avg} = 1050 \text{ kg/m}^3$ .

$$2.19 \quad \nabla p = \frac{\partial p}{\partial x} \mathbf{i} + \frac{\partial p}{\partial y} \mathbf{j} + \frac{\partial p}{\partial z} \mathbf{k}$$

$$= -\rho a_x \mathbf{i} - \rho a_y \mathbf{j} - \rho a_z \mathbf{k} - \rho g \mathbf{k} = -\rho (a_x \mathbf{i} + a_y \mathbf{j} + a_z \mathbf{k}) - \rho g \mathbf{k} = -\rho \mathbf{a} - \rho \mathbf{g}$$

$$\therefore \underline{\nabla p = -\rho(\mathbf{a} + \mathbf{g})}$$

$$2.20 \quad p = p_{atm} [(T_0 - \alpha z) / T_0]^{g/\alpha R}$$

$$= 100 [(288 - 0.0065 \times 300) / 288]^{9.81/0.0065 \times 287} = \underline{96.49 \text{ kPa}}$$

$$p = p_{atm} - \rho g h = 100 - \frac{100}{0.287 \times 288} \times 9.81 \times 300 / 1000 = \underline{96.44 \text{ kPa}}$$

$$\% \text{ error} = \frac{96.44 - 96.49}{96.49} \times 100 = \underline{-0.052\%}$$

The density variation can be ignored over heights of 300 m or less.

$$2.21 \quad \Delta p = p - p_0 = p_{atm} \left( \frac{T_0 - \alpha z}{T_0} \right)^{g/\alpha R} - p_{atm}$$

$$= 100 \left[ \left( \frac{288 - 0.0065 \times 20}{288} \right)^{9.81/0.0065 \times 287} - 1 \right]$$

$$= -0.237 \text{ Pa or } \underline{-0.000237 \text{ kPa}}$$

This change is very small and can most often be ignored.

2.22 Equation 1.5.11 gives

$$2.1 \times 10^9 = \rho \frac{dp}{d\rho}, \text{ where } dp = \rho g dh.$$

Therefore,

$$\rho g dh = \frac{2.1 \times 10^9}{\rho} d\rho$$

or, 
$$\frac{d\rho}{\rho^2} = \frac{9.81}{2.1 \times 10^9} dh = 4.67 \times 10^{-9} dh$$

Integrating from  $\rho_0 = 1030 \text{ kg/m}^3$ , we get

$$\int_{1030}^{\rho} \frac{d\rho}{\rho^2} = 4.67 \times 10^{-9} \int_0^h dh$$

or, 
$$\left( \frac{1}{1030} - \frac{1}{\rho} \right) = 4.67 \times 10^{-9} h$$

or, 
$$\rho = \frac{1}{9.71 \times 10^{-4} - 4.67 \times 10^{-9} h}$$

Hence,

$$p = \int_0^h \rho g dh = g \frac{1}{4.67 \times 10^{-9}} \ln \left( \frac{9.71 \times 10^{-4}}{9.71 \times 10^{-4} - 4.67 \times 10^{-9} h} \right)$$

$$p = 2.1 \times 10^9 \ln \left( \frac{1}{1 - 4.81 \times 10^{-6} h} \right) = p_{\text{accurate}}$$

Now, assuming  $\rho = \text{constant} = 1030 \text{ kg/m}^3$ , we have

$$p = \rho g h = 1030 \times 9.81 \times h = 10104h = p_{\text{estimate}}$$

a)  $h = 450 \text{ m}$ :  $p_{\text{accurate}} = 4550 \text{ kPa}$

$$p_{\text{estimate}} = 4547 \text{ kPa}$$

$$\% \text{ error} = \frac{4547 - 4550}{4550} \times 100 = -0.076$$

b)  $h = 1500 \text{ m}$ :  $p_{\text{accurate}} = 15206 \text{ kPa}$

$$p_{\text{estimate}} = 15156 \text{ kPa}$$

$$\% \text{ error} = \frac{15156 - 15206}{15206} \times 100 = -0.33$$

c)  $h = 4500 \text{ m}$ :  $p_{\text{accurate}} = 45954 \text{ kPa}$

$$p_{\text{estimate}} = 45469 \text{ kPa}$$

$$\% \text{ error} = \frac{45469 - 45954}{45954} \times 100 = -1.1$$

2.23 Use the result of Example 2.2:  $p = 101 e^{-gz/RT}$ .

a)  $p = 101 e^{-9.81 \times 10\,000/287 \times 273} = \underline{28.9 \text{ kPa}}$ .

b)  $p = 101 e^{-9.81 \times 10\,000/287 \times 288} = \underline{30.8 \text{ kPa}}$ .

c)  $p = 101 e^{-9.81 \times 10\,000/287 \times 258} = \underline{26.9 \text{ kPa}}$ .

2.24 Use Eq. 2.4.8:  $p = 101(1 - 0.0065z/288)^{\frac{9.81}{0.0065 \times 287}}$ .

a)  $z = 3000$ .  $\therefore p = \underline{69.9 \text{ kPa}}$ .

b)  $z = 6000$ .  $\therefore p = \underline{47.0 \text{ kPa}}$ .

c)  $z = 9000$ .  $\therefore p = \underline{30.6 \text{ kPa}}$ .

d)  $z = 11\,000$ .  $\therefore p = \underline{22.5 \text{ kPa}}$ .

2.25 Use the result of Example 2.2:  $\frac{p}{p_0} = e^{-gz/RT}$   $\therefore T = 253 \text{ k}$   $R = 287 \text{ J/kg k}$

$$\ln \frac{p}{p_0} = -\frac{gz}{RT} \quad \ln \left( \frac{7}{101.325 \times 10^3} \right) = -\frac{9.81z}{287 \times 253} \quad \Rightarrow z = 70,900 \text{ m.}$$

### Manometers

2.26  $p = \gamma h = (13.6 \times 9810) \times 0.25 = 33\,350 \text{ Pa}$  or  $\underline{33.35 \text{ kPa}}$ .

2.27 a)  $p = \gamma h$ .  $450\,000 = (13.6 \times 9810) h$ .  $\therefore h = \underline{3.373 \text{ m}}$

b)  $p + 11.78 \times 1.5 = (13.6 \times 9810) h$ . Use  $p = 450\,000$ , then  $h = \underline{3.373 \text{ m}}$

The % error is  $\underline{0.000\%}$ .

2.28 Referring to Fig. 2.6a, the pressure in the pipe is  $p = \rho gh$ . If  $p = 2400 \text{ Pa}$ , then

$$2400 = \rho gh = \rho \times 9.81h \quad \text{or} \quad \rho = \frac{2400}{9.81h}$$

a)  $\rho = \frac{2400}{9.81 \times 0.36} = 680 \text{ kg/m}^3$ .  $\therefore \underline{\text{gasoline}}$

b)  $\rho = \frac{2400}{9.81 \times 0.272} = 899 \text{ kg/m}^3$ .  $\therefore \underline{\text{benzene}}$

c)  $\rho = \frac{2400}{9.81 \times 0.245} = 999 \text{ kg/m}^3$ .  $\therefore \underline{\text{water}}$

d)  $\rho = \frac{2400}{9.81 \times 0.154} = 1589 \text{ kg/m}^3$ .  $\therefore \underline{\text{carbon tetrachloride}}$

2.29 Referring to Fig. 2.6a, the pressure is  $p = \rho_w gh = \frac{1}{2} \rho_a V^2$ . Then  $V^2 = \frac{2\rho_w gh}{\rho_a}$ .

a)  $V^2 = \frac{2 \times 1000 \times 9.81 \times 0.06}{1.23} = 957$ .  $\therefore V = \underline{30.9 \text{ m/s}}$

b)  $V^2 = \frac{2 \times 1000 \times 9.81 \times 0.075}{1.23} = 1196.34$ .  $\therefore V = \underline{34.6 \text{ m/s}}$

c)  $V^2 = \frac{2 \times 1000 \times 9.81 \times 0.1}{1.23} = 1595$ .  $\therefore V = \underline{39.9 \text{ m/s}}$

d)  $V^2 = \frac{2 \times 1000 \times 9.81 \times 0.125}{1.23} = 1994$ .  $\therefore V = \underline{44.6 \text{ m/s}}$

2.30 See Fig. 2.6b:  $p_1 = -\gamma h + \gamma_2 H$ .

$$p_1 = -0.86 \times 1000 \times 9.81 \times 0.125 + 13.6 \times 1000 \times 9.81 \times 0.24$$

$$= 30695 \text{ Pa} = 30.96 \text{ kPa.}$$

2.31  $p = p_0 + \rho_1 gh_1 + \rho_2 gh_2 + \rho_3 gh_3 + \rho_4 gh_4$

$$= 3200 + 917 \times 9.81 \times 0.2 + 1000 \times 9.81 \times 0.1 + 1258 \times 9.81 \times 0.15 + 1593 \times 9.81 \times 0.18$$

$$= 10\,640 \text{ Pa} \quad \text{or} \quad \underline{10.64 \text{ kPa}}$$

2.32  $p_1 - p_4 = (p_1 - p_2) + (p_2 - p_3) + (p_3 - p_4)$  (Use  $\Delta p = \rho g \Delta h$ )

$$40\,000 - 16\,000 = 1000 \times 9.81(-0.2) + 13\,600 \times 9.81 \times H + 920 \times 9.81 \times 0.3$$

$$\therefore H = 0.1743 \text{ m} \quad \text{or} \quad \underline{17.43 \text{ cm}}$$

2.33  $p_1 - p_4 = (p_1 - p_2) + (p_2 - p_3) + (p_3 - p_4)$  (Use  $\Delta p = \rho g \Delta h$ )

$$p_o - p_w = 900 \times 9.81(-0.2) + 13\,600 \times 9.81(-0.1) + 1000 \times 9.81 \times 0.15$$

$$= -12\,300 \text{ Pa} \quad \text{or} \quad \underline{-12.3 \text{ kPa}}$$

2.34  $p_1 - p_5 = (p_1 - p_2) + (p_2 - p_3) + (p_3 - p_4) + (p_4 - p_5)$

$$p_1 = 9810(-0.02) + 13\,600 \times 9.81(-0.04) + 9810(-0.02) + 13\,600 \times 9.81 \times 0.16$$

$$= 15\,620 \text{ Pa} \quad \text{or} \quad \underline{15.62 \text{ kPa}}$$

2.35  $p_w + 9810 \times 0.15 - 13.6 \times 9810 \times 0.1 - 0.68 \times 9810 \times 0.2 + 0.86 \times 9810 \times 0.15 = p_o$

$$\therefore p_w - p_o = 11\,940 \text{ Pa} \quad \text{or} \quad \underline{11.94 \text{ kPa.}}$$

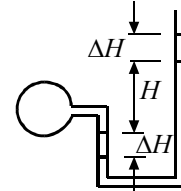
2.36  $p_w - 9810 \times 0.12 - 0.68 \times 9810 \times 0.1 + 0.86 \times 9810 \times 0.1 = p_o$

$$\text{With } p_w = 15\,000, \quad p_o = 14\,000 \text{ Pa} \quad \text{or} \quad \underline{14.0 \text{ kPa.}}$$

- 2.37 a)  $p + 9810 \times 2 = 13.6 \times 9810 \times 0.1$ .  $\therefore p = -6278 \text{ Pa}$  or -6.28 kPa.  
 b)  $p + 9810 \times 0.8 = 13.6 \times 9810 \times 0.2$ .  $\therefore p = 18\,835 \text{ Pa}$  or 18.84 kPa.  
 c)  $p + 9810 \times 1.8 = 13.6 \times 9810 \times 0.1$ .  $\therefore p = -4316 \text{ Pa} = -4.316 \text{ kPa.}$   
 d)  $p + 9810 \times 0.6 = 13.6 \times 9810 \times 0.2$ .  $\therefore p = 20797 \text{ Pa} = 20.8 \text{ kPa.}$

2.38  $p - 9810 \times 4 + 13.6 \times 9810 \times 0.16 = 0$ .  $\therefore p = 17\,890 \text{ Pa}$  or 17.89 kPa.

2.39  $8200 + 9810 \times 0.25 = 1.59 \times 9810 \times H$ .  $\therefore H = 0.683 \text{ m}$   
 $H_{new} = 0.683 + 0.273 = 0.956 \text{ m.}$   $\Delta H = \frac{0.273}{2} = 0.1365.$   
 $p + 9810 (0.25 + 0.1365) = 1.59 \times 9810 \times 0.956.$   
 $\therefore p = 11\,120 \text{ Pa}$  or 11.12 kPa.



2.40  $p + 9810 \times 0.05 + 1.59 \times 9810 \times 0.07 - 0.8 \times 9810 \times 0.1 = 13.6 \times 9810 \times 0.05.$   
 $\therefore p = 5873 \text{ Pa}$  or 5.87 kPa.

Note: In our solutions we usually retain 3 significant digits in the answers (if a number starts with “1” then 4 digits are retained). In most problems a material property is used, i.e.,  $S = 1.59$ . This is only 3 significant digits!  $\therefore$  only 3 are usually retained in the answer!

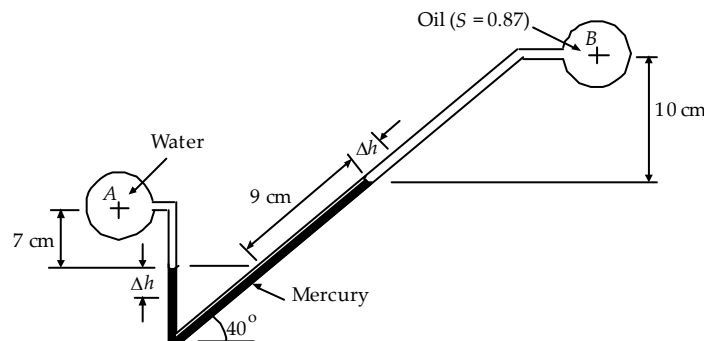
2.41 The equation for the manometer is

$$p_A + \gamma_{\text{water}} \times 0.07 = p_B + \gamma_{\text{oil}} \times 0.1 + \gamma_{\text{HG}} \times 0.09 \sin 40$$

Solve for  $p_B$ :

$$\begin{aligned} p_B &= p_A + \gamma_{\text{water}} \times 0.07 - \gamma_{\text{HG}} \times 0.09 \sin 40 - \gamma_{\text{oil}} \times 0.1 \\ &= p_A + \gamma_{\text{water}} \times 0.07 - 13.6 \gamma_{\text{water}} \times 0.09 \sin 40 - 0.87 \gamma_{\text{water}} \times 0.1 \\ &= p_A + (0.07 - 13.6 \times 0.09 \sin 40 - 0.87 \times 0.1) \times \gamma_{\text{water}} \\ &= 10 \text{ kPa} + (0.07 - 13.6 \times 0.09 \sin 40 - 0.87 \times 0.1) \times 9.81 \text{ kN/m}^3 = 2.11 \text{ kPa} \end{aligned}$$

2.42 The distance the mercury drops on the left equals the distance along the tube that the mercury rises on the right. This is shown in the sketch.



From the previous problem we have

$$(p_B)_1 = p_A + \gamma_{\text{water}} \times 0.07 - \gamma_{\text{HG}} \times 0.09 \sin 40 - \gamma_{\text{oil}} \times 0.1 \quad (1)$$

For the new condition

$$(p_B)_2 = p_A + \gamma_{\text{water}} \times (0.07 + \Delta h) - \gamma_{\text{HG}} \times 0.11 \sin 40 - \gamma_{\text{oil}} \times (0.1 - \Delta h \sin 40) \quad (2)$$

where  $\Delta h$  in this case is calculated from the new manometer reading as

$$\Delta h + \Delta h / \sin 40 = 11 - 9 \Rightarrow \Delta h = 0.783 \text{ cm}$$

Subtracting Eq.(1) from Eq.(2) yields

$$(p_B)_2 - (p_B)_1 = \gamma_{\text{water}} \times (\Delta h) - \gamma_{\text{HG}} \times 0.02 \sin 40 - \gamma_{\text{oil}} \times (-\Delta h \sin 40)$$

Substituting the values of  $\Delta h$  and  $(p_B)_1$  gives  $(p_B)_2$

$$\begin{aligned} (p_B)_2 &= 2.11 + [(0.00783) - 13.6 \times 0.02 \sin 40 - 0.87 \times (-0.00783 \sin 40)] \times 9.81 \\ &= \underline{0.52 \text{ kPa}} \end{aligned}$$

- 2.43 Before pressure is applied the air column on the right is 120cm high. After pressure is applied, it is  $(1.2 - H/2)$  m high. For an isothermal process  $p_1 V_1 = p_2 V_2$  using absolute pressures. Thus,

$$101.3 \times 1.2 \times A = p_2 (1.2 - H/2) A \quad \text{or} \quad p_2 = \frac{121.56}{(1.2 - H/2)}$$

From a pressure balance on the manometer (pressures in kPa):

$$207 + 101.3 = 13.6 \times \frac{9810}{1000} + \frac{121.56}{1.2 - H/2},$$

$$\text{or} \quad H^2 - 4.71H + 3.73 = 0 \quad \therefore H = 1.01 \text{ m} \quad \text{or} \quad 3.7 \text{ m.}$$

- 2.44  $p_1 - p_5 = (p_1 - p_2) + (p_2 - p_3) + (p_3 - p_4) + (p_4 - p_5)$   
 $4000 = 9800(0.16 - 0.22) + 15\,600(0.10 - 0.16) + 133\,400H + 15\,600(0.07 - H)$   
 $\therefore H = 0.0376 \text{ m} \text{ or } \underline{37.6 \text{ mm}}$

- 2.45 
$$\frac{\Delta H}{\Delta p_1} = \frac{2D^2 / d^2}{-\gamma_1 + 2\gamma_2 + 2(\gamma_3 - \gamma_2)D^2 / d^2}$$

$$= \frac{2(0.1/0.005)^2}{-9800 + 2 \times 15\,600 + 2(133\,400 - 15\,600)(0.1/0.005)^2} = 8.487 \times 10H^{-6}$$

$$\therefore \Delta H = 8.487 \times 10^{-6} \times 400 = 0.0034 \text{ m} \quad \text{or} \quad 3.4 \text{ mm}$$

- 2.46  $p_1 - p_4 = (p_1 - p_2) + (p_2 - p_3) + (p_3 - p_4)$  ( $p_{\text{oil}} = 14.0 \text{ kPa}$  from No. 2.30)  
 $15\,500 - 14\,000 = 9800(0.12 + \Delta z) + 680(0.1 - 2\Delta z) + 860(-0.1 - \Delta z)$   
 $\therefore \Delta z = 0.0451 \text{ m} \text{ or } \underline{45.1 \text{ mm}}$



- 2.47 a)  $p_{air} = -6250 + 625 = -5620$  Pa.  
 $-5620 + 9800(2 + \Delta z) - 13\,600 \times 9.81(0.1 + 2\Delta z) = 0. \quad \therefore \Delta z = 0.0025.$   
 $\therefore h = 0.1 + 2\Delta z = 0.15$  m or 15 cm
- b)  $p_{air} = 18\,800 + 1880 = 20\,680$  Pa.  
 $20\,680 + 9800(0.8 + \Delta z) - 13\,600 \times 9.81(0.2 + 2\Delta z) = 0. \quad \therefore \Delta z = 0.00715$  m  
 $\therefore h = 0.2 + 2\Delta z = 0.214$  or 21.4 cm
- c)  $p_{air} = -4316 + 431.6 = -3884$  Pa.  
 $-3884 + 9800(1.8 + \Delta z) - 13600 \times 9.81(0.1 + 2\Delta z) = 0. \quad \Rightarrow \Delta z = 1.61 \times 10^{-3}$  m  
 $= 161$  mm.
- d)  $p_{air} = 20800 + 2080 = 22880$  Pa  
 $22880 + 9800(0.6 + \Delta z) - 13600 \times 9.81(0.2 + 2\Delta z)$   
 $\Rightarrow \Delta z = 8.1 \times 10^{-3}$  m = 8.1 mm.

**Forces on Plane Areas**

- 2.48  $F = \bar{\gamma} A = 9810 \times 10 \times \pi \times 0.3^2/4 = \underline{6934}$  N.
- 2.49  $\left(2 \times \frac{5}{3}\right) \times P = \frac{1}{3} \times \left(2 \times \frac{5}{3}\right) \times \left[9800 \times 1 \times 3 \times \left(2 \times \frac{5}{3}\right)\right]. \quad \therefore P = 32\,670$  N
- a)  $F = p_c A = 9800 \times 2 \times 4^2 = 313\,600$  N or 313.6 kN
- b)  $F = p_c A = 9800 \times 1 \times (2 \times 4) + 9800 \times \frac{2}{3} \times 2 + 9800 \times \frac{2}{3} \times 1 = 98\,000$  N or 98.0 kN
- c)  $F = p_c A = 9800 \times 1 \times 2 \times 4 \times \sqrt{2} = 110\,900$  N or 110.9 kN
- d)  $F = p_c A = 9800 \times 1 \times 2 \times 4/0.866 = 90\,500$  N or 90.5 kN
- 2.50 For saturated ground, the force on the bottom tending to lift the vault is  
 $F = p_c A = 9800 \times 1.5 \times (2 \times 1) = 29\,400$  N  
 The weight of the vault is approximately  
 $W = \rho g V_{\text{walls}} = 2400 \times 9.81 [2(2 \times 1.5 \times 0.1) + 2(2 \times 1 \times 0.1) + 20(.8 \times 1.3 \times 0.1)] = 28\,400$  N.  
 The vault will tend to rise out of the ground.
- 2.51  $F = p_c A = 6660 \times 2 \times \pi \times 2^2 = 167\,400$  N or 167.4 kN  
 Find  $\gamma$  in Table B.5 in the Appendix.
- 2.52 a)  $F = p_c A = 9800 (10 - 2.828/3) (2.828 \times 2/2) = 251\,000$  N or 251 kN  
 where the height of the triangle is  $(3^2 - 1^2)^{1/2} = 2.828$  m.
- b)  $F = p_c A = 9800 \times 10 (2.828 \times 2/2) = 277\,100$  N or 277.1 kN
- c)  $F = p_c A = 9800 (10 - 2.828 \times 0.866/3) (2.828 \times 2/2) = 254\,500$  N or 254.5 kN

$$2.53 \quad a) \quad F = \gamma \bar{h} A = 9810 \times 8.2 \times \frac{(2.4 \times 1.8)}{2} = 174,995 \text{ N.}$$

$$y_p = 8.2 + \frac{1.8 \times (2.4)^3 / 36}{8.2 \times 2.16} = 8.24 \text{ m.} \quad \therefore y = 9 - 8.24 = 0.76 \text{ m.}$$

$$\text{Now, } \frac{2.4}{1.64} = \frac{0.9}{x} \Rightarrow x = 0.61 \text{ m.}$$

Thus, the force acts at (0.61m, 0.76 m).

$$b) \quad F = 9810 \times 9 \times \frac{(2.4 \times 1.8)}{2} = 192,070 \text{ N. The centroid is the center of pressure.}$$

$$y = 0.8 \text{ m.} \quad \Rightarrow \quad \frac{2.4}{1.6} = \frac{0.9}{x} \quad \Rightarrow \quad x = 0.6 \text{ m.}$$

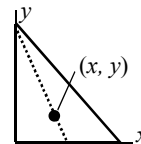
Thus, the location of force is (0.6m, 0.8m).

$$c) \quad F = 9810(9 - 0.8 \times 0.707) \times \frac{(2.4 \times 1.8)}{2} = 179,997 \text{ N.}$$

$$y_p = 11.93 + \frac{1.8 \times (2.4)^3 / 36}{11.93 \times 2.16} = 11.97 \text{ m.} \quad y = 12.74 - 11.97 = 0.77 \text{ m}$$

$$\text{Now, } \frac{2.4}{1.63} = \frac{0.9}{x} \Rightarrow x = 0.61 \text{ m.}$$

Thus, the location is (0.61m, 0.77 m).



$$2.54 \quad a) \quad F = \gamma \bar{h} A = 9810 \times 6 \times \pi 2^2 = 739\,700 \text{ N or } \underline{739.7 \text{ kN.}}$$

$$y_p = \bar{y} + \frac{\bar{I}}{A\bar{y}} = 6 + \frac{\pi \times 2^4}{4(4\pi \times 6)} = 6.167 \text{ m.} \quad \therefore (x, y)_p = \underline{(0, -0.167) \text{ m}}$$

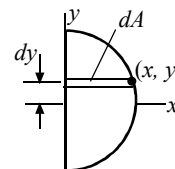
$$b) \quad F = \gamma \bar{h} A = 9810 \times 6 \times 2\pi = 369\,800 \text{ N or } \underline{369.8 \text{ kN.}}$$

$$y_p = 6 + \frac{\pi \times 2^4 / 8}{2\pi \times 6} = 6.167 \text{ m.} \quad x^2 + y^2 = 4$$

$$x_p F = \int \frac{x}{2} p dA = \frac{\gamma}{2} \int_{-2}^2 x(6-y)x dy = \frac{\gamma}{2} \int_{-2}^2 (4-y^2)(6-y) dy.$$

$$\therefore x_p \gamma 6 \times 2\pi = \frac{\gamma}{2} \int_{-2}^2 (24 - 4y - 6y^2 + y^3) dy = 32\gamma. \quad \therefore x_p = 0.8488 \text{ m}$$

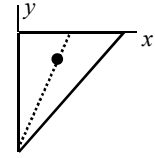
$$\therefore (x, y)_p = \underline{(0.8488, -0.167) \text{ m}}$$



$$c) \quad F = 9810 \times (4 + 4/3) \times 6 = 313\,900 \text{ N or } \underline{313.9 \text{ kN.}}$$

$$y_p = 5.333 + \frac{3 \times 4^3 / 36}{5.333 \times 6} = 5.500 \text{ m. } \therefore y = -1.5$$

$$4/2.5 = \frac{1.5}{x}. \quad \therefore x = 0.9375. \quad \therefore (x, y)_p = \underline{(0.9375, -1.5) \text{ m}}$$

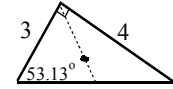


$$d) F = 9810 \times (4 + \frac{2}{3} \times 4 \sin 36.9^\circ) \times 6 = \underline{330\,000 \text{ N}}$$

$$y_p = 5.6 + \frac{5 \times 2.4^3 / 36}{6 \times 5.6} = 5.657 \text{ m. } \therefore y = 0.343 \text{ m}$$

$$3 \cos 53.13^\circ = 1.8, \quad 2.5 - 1.8 = 0.7, \quad 2.4 / 2.057 = .7 / x_1. \quad \therefore x_1 = 0.6.$$

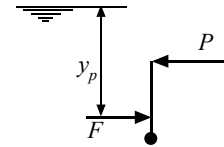
$$x = 1.8 + 0.6 = 2.4. \quad \therefore (x, y)_p = \underline{(2.4, 0.343) \text{ m.}}$$



$$2.55 \quad F = \gamma \bar{h} A = 9810 \times (1.8 + 1.5) \times (1.8 \times 3) = 176062 \text{ N.}$$

$$y_p = \bar{y} + \frac{\bar{I}}{\bar{y}A} = 3.3 + \frac{1.8 \times 3^3 / 12}{3.3 \times 1.8 \times 3} = 3.53 \text{ m.}$$

$$(4.8 - 3.53) 176062 = 3 \times P. \quad \Rightarrow P = 74,533 \text{ N.}$$



$$2.56 \quad F = \gamma \bar{h} A = 9810 \times 6 \times 20 = 1.777 \times 10^6 \text{ N, or } 1177 \text{ kN.}$$

$$y_p = \bar{y} + \frac{\bar{I}}{A\bar{y}} = 7.5 + \frac{4 \times 5^3 / 12}{7.5 \times 20} = 7.778 \text{ m.}$$

$$(10 - 7.778) 1177 = 5 P. \quad \therefore P = \underline{523 \text{ kN.}}$$

$$2.57 \quad a) F = \gamma \bar{h} A = 9810 \times 8 \times 20 = 1.57 \times 10^6 \text{ N} = 1570 \text{ kN}$$

$$y_p = \bar{y} + \frac{\bar{I}}{A\bar{y}} = 9.5 + \frac{(4 \times 5^3) / 12}{20 \times 9.5} = 9.72 \text{ m}$$

$$(12 - 9.72) 1570 = 5P$$

$$P = \boxed{716 \text{ kN}}$$

$$b) F = \gamma \bar{h} A = 9810 \times 10 \times 20 = 1.57 \times 10^6 \text{ N} = 1962 \text{ kN}$$

$$y_p = \bar{y} + \frac{\bar{I}}{A\bar{y}} = 11.5 + \frac{(4 \times 5^3) / 12}{20 \times 11.5} = 11.68 \text{ m}$$

$$(14 - 11.68) 1962 = 5P$$

$$P = \boxed{910 \text{ kN}}$$

$$c) F = \gamma \bar{h} A = 9810 \times 12 \times 20 = 1.57 \times 10^6 \text{ N} = 2354 \text{ kN}$$

$$y_p = \bar{y} + \frac{\bar{I}}{A\bar{y}} = 13.5 + \frac{(4 \times 5^3)/12}{20 \times 13.5} = 13.65 \text{ m}$$

$$(16 - 13.65)2354 = 5P$$

$$P = \boxed{1106 \text{ kN}}$$

$$2.58 \quad y_p = \bar{y} + \frac{\bar{I}}{A\bar{y}} = \frac{H}{2} + \frac{bH^3/12}{bH \times H/2} = \frac{H}{2} + \frac{H}{6} = \frac{2}{3}H. \quad y_p \text{ is measured from the surface.}$$

$$\therefore \text{From the bottom, } H - y_p = H - \frac{2}{3}H = \frac{1}{3}H.$$

Note: This result is independent of the angle  $\alpha$ , so it is true for a vertical area or a sloped area.

$$2.59 \quad F = \gamma \frac{1}{2} l \sin 40^\circ \times 3l. \quad F \times \frac{l}{3} = (l+2)P \sin 40^\circ. \quad \therefore \gamma l^3 = 2(l+2)P.$$

$$\text{a) } 9810 \times 2^3 = 2(2+2)P. \quad \therefore P = \underline{9810 \text{ N}}$$

$$\text{b) } 9810 \times 4^3 = 2(4+2)P. \quad \therefore P = \underline{52\,300 \text{ N}}$$

$$\text{c) } 9810 \times 5^3 = 2(5+2)P. \quad \therefore P = \underline{87\,600 \text{ N}}$$

$$2.60 \quad h = \sqrt{1.2^2 - 0.4^2} = 1.1314 \text{ m. } A = 1.2 \times 1.1314 + 0.4 \times 1.1314 = 1.8102 \text{ m}^2$$

$$\text{Use 2 forces: } F_1 = \gamma h_c A_1 = 9800 \times 0.5657 \times (1.2 \times 1.1314) = 7527 \text{ N}$$

$$F_2 = \gamma h_c A_2 = 9800 \times \frac{1.1314}{3} \times (0.4 \times 1.1314) = 1673 \text{ N}$$

$$y_{p1} = \frac{2}{3}(1.1314). \quad y_{p2} = \bar{y} + \frac{\bar{I}_2}{A_2 \bar{y}} = \frac{1.1314}{3} + \frac{0.4 \times 1.1314^3 / 36}{0.4 \times (1.1314/2) \times (1.1314/3)} = 0.5657 \text{ m}$$

$$\Sigma M_{\text{hinge}} = 0: \quad 7527 \times \frac{1.1314}{3} + 1673 \times (1.1314 - 0.5657) - 1.1314P = 0. \quad \therefore P = \underline{3346 \text{ N.}}$$

2.61 To open, the resultant force must be just above the hinge, i.e.,  $y_p$  must be just less than  $h$ . Let  $y_p = h$ , the condition when the gate is about to open:

$$\bar{y} = (h+H)/3, \quad A = (h+H)^2, \quad \bar{I} = [2(h+H)](h+H)^3/36$$

$$\therefore y_p = \frac{h+H}{3} + \frac{2(h+H)^4/36}{(h+H)^2(h+H)/3} = \frac{h+H}{3} + \frac{h+H}{6} = \frac{h+H}{2}$$

$$\text{a) } h = \frac{h+H}{2}. \quad \therefore h = H = \underline{0.9 \text{ m}}$$

$$\text{b) } h = H = \underline{1.2 \text{ m}}$$

$$\text{c) } h = H = \underline{1.5 \text{ m}}$$

2.62 The gate is about to open when the center of pressure is at the hinge.

$$a) y_p = 1.2 + H = (1.8/2 + H) + \frac{b \times 1.8^3 / 12}{(0.9 + H)1.8b} \quad \therefore H = \underline{0}$$

$$b) y_p = 1.2 + H = (2.0/2 + H) + \frac{b \times 2^3 / 12}{(1 + H)2b} \quad \therefore H = \underline{0.6667 \text{ m}}$$

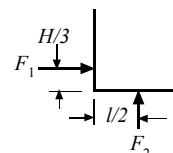
$$c) y_p = 1.2 + H = (2.2/2 + H) + \frac{b \times 2.2^3 / 12}{(1.1 + H)2.2b} \quad \therefore H = \underline{2.933 \text{ m}}$$

$$2.63 \quad F_1 = \gamma \frac{H}{2} \times bH = \frac{1}{2} \gamma bH^2$$

$$F_2 = \gamma H \times \ell b = \gamma b \ell H$$

$$\frac{1}{2} \gamma bH^2 \times \frac{H}{3} = \gamma b \ell H \times \frac{\ell}{2} \quad \therefore H = \sqrt{3} \ell$$

$$a) H = \sqrt{3} \times 2 = \underline{3.464 \text{ m}} \quad b) H = \underline{1.732 \text{ m}} \quad c) H = \underline{3.12 \text{ m}} \quad d) H = \underline{1.56 \text{ m}}$$



2.64 A free-body-diagram of the gate and block is sketched.

Sum forces on the block:

$$\Sigma F_y = 0 \quad \therefore W = T + F_B$$

where  $F_B$  is the buoyancy force which is given by

$$F_B = \gamma [\pi R^2 (3 - H)]$$

Take moments about the hinge:

$$T \times 3.5 = F_H \times (3 - y_p)$$

where  $F_H$  is the hydrostatic force acting on the gate. It is, using  $\bar{h} = 1.5 \text{ m}$  and  $A = 2 \times 3 = 6 \text{ m}^2$ ,

$$F_H = \gamma \bar{h} A = (9.81 \text{ kN/m}^3)(1.5 \text{ m} \times 6 \text{ m}^2) = 88.29 \text{ kN}$$

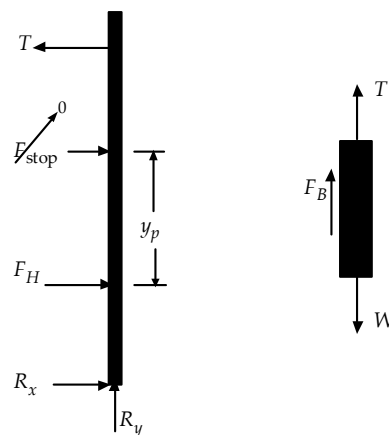
From the given information,

$$y_p = \bar{y} + \frac{\bar{I}}{\bar{y}A} = 1.5 + \frac{2(3^3)/12}{1.5 \times 6} = 2 \text{ m}$$

$$\therefore T = \frac{88.29 \times (3 - 2)}{3.5} = 25.23 \text{ kN}$$

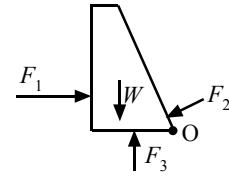
$$F_B = W - T = 70 - 25.23 = 44.77 \text{ kN} \quad \therefore \gamma \pi R^2 (3 - H) = 44.77$$

$$H = 3 \text{ m} - \frac{44.77 \text{ kN}}{(9.81 \text{ kN/m}^3) \pi (1 \text{ m})^2} = \underline{1.55 \text{ m}}$$



Assume 1 m deep

2.65 The dam will topple if the moment about “O” of  $F_1$  and  $F_3$  exceeds the restoring moment of  $W$  and  $F_2$ .



a)  $W = (2.4 \times 9810)(6 \times 50 + 24 \times 50/2) = 21.19 \times 10^6 \text{ N}$

$$d_w = \frac{300 \times 27 + 600 \times 16}{300 + 600} = 19.67 \text{ m. (} d_w \text{ is from O to } W\text{.)}$$

$$F_2 = 9810 \times 5 \times 11.09 = 0.544 \times 10^6 \text{ N. } d_2 = \frac{11.09}{3} = 3.697 \text{ m.}$$

$$F_1 = 9810 \times \frac{45}{2} \times 45 = 9.933 \times 10^6 \text{ N. } d_1 = 15 \text{ m. (} d_1 \text{ is from O to } F_1\text{.)}$$

$$F_3 = 9810 \times \frac{45 + 10}{2} \times 30 = 8.093 \times 10^6 \text{ N. } d_3 = \frac{2.943 \times 15 + 5.150 \times 20}{2.943 + 5.150} = 18.18 \text{ m.}$$

$$\left. \begin{aligned} Wd_w + F_2d_2 &= 418.8 \times 10^6 \text{ N} \cdot \text{m} \\ F_1d_1 + F_3d_3 &= 296.1 \times 10^6 \text{ N} \cdot \text{m} \end{aligned} \right\} \therefore \underline{\text{will not topple.}}$$

b)  $W = (2.4 \times 9810)(6 \times 65 + 65 \times 12) = 27.55 \times 10^6 \text{ N.}$

$$d_w = \frac{390 \times 27 + 780 \times 16}{390 + 780} = 19.67 \text{ m.}$$

$$F_2 \cong 0.54 \times 10^6 \text{ N. } d_2 \cong 3.70 \text{ m.}$$

$$F_1 = 9810 \times 30 \times 60 = 17.66 \times 10^6 \text{ N. } d_1 = 20 \text{ m.}$$

$$F_3 = 9810 \times \frac{60 + 10}{2} \times 30 = 10.3 \times 10^6 \text{ N. } d_3 = \frac{2.943 \times 15 + 7.358 \times 20}{2.943 + 7.358} = 18.57 \text{ m.}$$

$$\left. \begin{aligned} Wd_w + F_2d_2 &= 543.9 \times 10^6 \text{ N} \cdot \text{m} \\ F_1d_1 + F_3d_3 &= 544.5 \times 10^6 \text{ N} \cdot \text{m} \end{aligned} \right\} \therefore \underline{\text{it will topple.}}$$

c) Since it will topple for  $H = 60$ , it certainly will topple if  $H = 75$  m.

2.66 The dam will topple if there is a net clockwise moment about “O.”

a)  $W = W_1 + W_2$ .  $W_1 = (1.8 \times 12.9 \times 1) \times 9810 \times 2.4 = 546.7 \text{ kN.}$

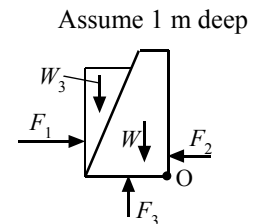
$$W_2 = 7.2 \times \frac{12.9}{2} \times 9810 \times 2.4 = 1093.4 \text{ kN.}$$

$$\begin{aligned} W_3 &= (12 \times 6.7/2) \times 9810 \\ &= 412.02 \text{ kN @ } 6.27 \text{ m.} \end{aligned}$$

$$F_1 = 9810 \times 6 \times (12 \times 1) = 706.32 \text{ kN @ } 4 \text{ m.}$$

$$F_2 = 9810 \times 1.5 \times (3 \times 1) = 44.14 \text{ kN @ } 1 \text{ m.}$$

$$F_3 = \begin{cases} F_{p1} = 264.9 \text{ kN @ } 4.5 \text{ m} \\ F_{p2} = 397.3 \text{ kN @ } 6 \text{ m} \end{cases}$$



Net moment about O:

$$\begin{aligned}\Sigma M_O &= 706.32 \times 4 + 264.9 \times 4.5 + 397.3 \times 6 - 546.7 \times 0.9 - 1093.4 \times 4.2 \\ &\quad - 412.02 \times 6.27 - 44.14 \times 1 \\ &= -851.2 \text{ kN-m} < 0 \text{ and hence the dam won't tip.}\end{aligned}$$

$$\text{b) } W_1 = (1.8 \times 18.9 \times 1) \times 9810 \times 2.4 = 801 \text{ kN.}$$

$$W_2 = 7.2 \times \frac{18.9}{2} \times 9810 \times 2.4 = 1602 \text{ kN.}$$

$$W_3 = 18 \times \frac{6.86}{2} \times 9810 = 605.7 \text{ kN.}$$

$$F_1 = 9810 \times 9 \times 18 = 1589 \text{ kN.}$$

$$F_2 = 9810 \times 1.5 \times 3 = 44.14 \text{ kN.}$$

$$F_{p_1} = 9810 \times 3 \times 9 = 265 \text{ kN}; \quad F_{p_2} = 9810 \times 15 \times \frac{9}{2} = 662.2 \text{ kN.}$$

Net moment about O:

$$\begin{aligned}\Sigma M_O &= 1589 \times 6 + 265 \times 4.5 + 662.2 \times 6 - 801 \times 0.9 - 1602 \times 4.2 - 605.7 \times 6.37 \\ &= 3392 \text{ kN-m} > 0\end{aligned}$$

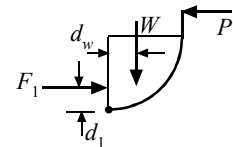
Hence, the dam will tip.

c) Since it will topple for  $H = 18 \text{ m.}$ , it will also topple for  $H = 24 \text{ m.}$

### Forces on Curved Surfaces

$$2.67 \quad \Sigma M_{\text{hinge}} = 0. \quad 2.5P - d_w \times W - d_1 \times F_1 = 0.$$

$$\therefore P = \frac{1}{2.5} \left[ \frac{2}{3} \times 9800 \times 1 \times 8 + \frac{4 \times 2}{3\pi} \times 9800 \times \frac{\pi \times 2^2}{4} \times 4 \right] = \underline{62\,700 \text{ N}}$$



Note: This calculation is simpler than that of Example 2.7. Actually, We could have moved the horizontal force  $F_H$  and a vertical force  $F_V$  (equal to  $W$ ) simultaneously to the center of the circle and then  $2.5P = 2F_H = 2F_V$ . This was outlined at the end of Example 2.7.

2.68 Since all infinitesimal pressure forces pass thru the center, we can place the resultant forces at the center. Since the vertical components pass thru the bottom point, they produce no moment about that point. Hence, consider only horizontal forces:

$$(F_H)_{\text{water}} = 9810 \times 2 \times (4 \times 10) = 784\,800 \text{ N}$$

$$(F_H)_{\text{oil}} = 0.86 \times 9810 \times 1 \times 20 = 168\,700 \text{ N}$$

$$\Sigma M: 2P = 784.8 \times 2 - 168.7 \times 2. \quad \therefore \underline{P = 616.1 \text{ kN.}}$$

2.69 a) Place the resultant force  $\vec{F}_H + \vec{F}_V$  at the center of the circular arc.  $\vec{F}_H$  passes through the hinge showing that  $P = F_V$ .

$$P = F_V = 9810(6 \times 2 \times 4 + \pi \times 4) = 594\,200 \text{ N or } \underline{594.2 \text{ kN}}.$$

b) Repeating the process from part a:

$$P = F_V = 9810(9 \times 3 \times 4 + \pi \times 4) = \boxed{1180 \text{ kN}}$$

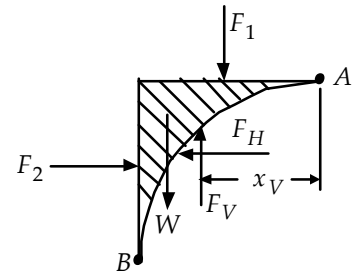
2.70 a) A free-body-diagram of the volume of water in the vicinity of the surface is shown. Force balances in the horizontal and vertical directions give:

$$F_H = F_2$$

$$F_V = W + F_1$$

where  $F_H$  and  $F_V$  are the horizontal and vertical components of the force acting on the water by the surface  $AB$ . Hence,

$$F_H = F_2 = (9.81 \text{ kN/m}^3)(8+1)(2 \times 4) = 706 \text{ kN}$$



The line of action of  $F_H$  is the same as that of  $F_2$ . Its distance from the surface is

$$y_p = \bar{y} + \frac{\bar{I}}{\bar{y}A} = 9 + \frac{4(2^3)/12}{9 \times 8} = 9.037 \text{ m}$$

To find  $F_V$  we find  $W$  and  $F_1$ :

$$W = \gamma V = (9.81 \text{ kN/m}^3) \left[ 2 \times 2 - \frac{\pi}{4}(2^2) \right] \times 4 = 33.7 \text{ kN}$$

$$F_1 = 9.81 \text{ kN/m}^3 (8 \times 2 \times 4) = 628 \text{ kN}$$

$$\therefore F_V = F_1 + W = 33.7 + 628 = 662 \text{ kN}$$

To find the line of action of  $F_V$ , we take moments at point  $A$ :

$$F_V \times x_V = F_1 \times d_1 + W \times d_2$$

where  $d_1 = 1 \text{ m}$ , and  $d_2 = \frac{2R}{3(4-\pi)} = \frac{2 \times 2}{3(4-\pi)} = 1.553 \text{ m}$ :

$$\therefore x_V = \frac{F_1 \times d_1 + W \times d_2}{F_V} = \frac{628 \times 1 + 33.7 \times 1.553}{662} = 1.028 \text{ m}$$

Finally, the forces  $F_H$  and  $F_V$  that act on the surface  $AB$  are equal and opposite to those calculated above. So, on the surface,  $F_H$  acts to the right and  $F_V$  acts downward.



b) If the water exists on the opposite side of the surface  $AB$ , the pressure distribution would be identical to that of Part (a). Consequently, the forces due to that pressure distribution would have the same magnitudes. The vertical force  $F_V = 662 \text{ N}$  would act upward and the horizontal force  $F_H = 706.3 \text{ N}$  would act to the left.

- 2.71 Place the resultant  $\mathbf{F}_H + \mathbf{F}_V$  at the circular arc center.  $\mathbf{F}_H$  passes thru the hinge so that  $P = F_V$ . Use the water that could be contained above the gate; it produces the same pressure distribution and hence the same  $F_V$ . We have

$$P = F_V = 9810 (6 \times 3 \times 4 + 9\pi) = 983\,700 \text{ N or } \underline{983.7 \text{ kN.}}$$

- 2.72 Place the resultant  $\mathbf{F}_H + \mathbf{F}_V$  at the center.  $\mathbf{F}_V$  passes thru the hinge

$$2 \times (9810 \times 1 \times 10) = 2.8 P. \quad \therefore P = 70\,070 \text{ N or } \underline{70.07 \text{ kN.}}$$

- 2.73 The incremental pressure forces on the circular quarter arc pass through the hinge so that no moment is produced by such forces. Moments about the hinge gives:

$$3 P = 0.9 W = 0.9 \times 400. \quad \therefore \underline{P = 120 \text{ N.}}$$

- 2.74 The resultant  $\mathbf{F}_H + \mathbf{F}_V$  of the unknown liquid acts thru the center of the circular arc.  $\mathbf{F}_V$  passes thru the hinge. Thus, we use only  $(F_H)_{oil}$ . Assume 1 m wide:

$$\Sigma M: \frac{R}{3} \left( 9810 \frac{R}{2} R \right) + \frac{4R}{3\pi} \left( 9800S \frac{\pi R^2}{4} \right) = R \left( \gamma_x \frac{R}{2} R \right). \quad \therefore \gamma_x = \underline{4580 \text{ N/m}^3}$$

- 2.75 The force of the water is only vertical  $(F_V)_w$ , acting thru the center. The force of the oil can also be positioned at the center:

$$\text{a) } P = (F_H)_o = (0.8 \times 9810) \times 0.3 \times 3.6 = \underline{8476 \text{ N.}}$$

$$\Sigma F_y = 0 = W + (F_V)_o - (F_V)_w$$

$$0 = S \times 9810\pi \times 0.6^2 \times 6 + \left( 0.36 - \frac{0.36\pi}{4} \right) \times 6 \times (0.8 \times 9810) - 9810\pi \times 0.18 \times 6$$

$$-9810 \times 0.8 \times 2 \times 0.6^2 \times 6 \quad \therefore S = \underline{0.955.}$$

$$\text{b) } \rho g V = W.$$

$$\Sigma F_y = 0 = W + (F_V)_o - (F_V)_w$$

$$\Rightarrow 0 = S \times 9810\pi \times 1^2 \times 10 + \left( 1^2 - \frac{\pi}{4} \right) \times 10 \times (0.8 \times 9810) - 9810\pi \times \left( \frac{1}{2} \right) \times 10$$

$$-9810 \times 0.8 \times 2 \times 1^2 \times 10. \quad \Rightarrow S = \underline{0.955.}$$

2.76 The pressure in the dome is

a)  $p = 60\,000 - 9810 \times 3 - 0.8 \times 9810 \times 2 = 14\,870 \text{ Pa}$  or  $14.87 \text{ kPa}$ .

The force is  $F = pA_{\text{projected}} = (\pi \times 3^2) \times 14.87 = \underline{420.4 \text{ kN}}$ .

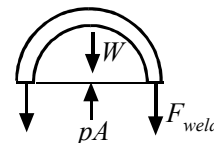
b) From a free-body diagram of the dome filled with oil:

$$F_{\text{weld}} + W = pA$$

Using the pressure from part (a):

$$F_{\text{weld}} = 14\,870 \times \pi \times 3^2 - (0.8 \times 9810) \times \frac{1}{2} \left( \frac{4}{3} \pi \times 3^3 \right) = -23\,400 \text{ N}$$

or  $\underline{-23.4 \text{ kN}}$



2.77 A free-body diagram of the gate and water is shown.

$$\frac{H}{3} F + d_w W = H \times P.$$

a)  $H = 2 \text{ m}$ .  $F = 9810 \times 1 \times 4 = 39\,240 \text{ N}$ .

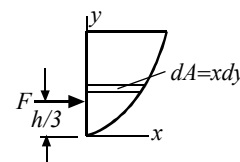
$$W = 9810 \int_0^2 2x dy = 9810 \int_0^2 2 \frac{y^{1/2}}{\sqrt{2}} dy = \frac{2 \times 9810}{\sqrt{2}} \frac{2^{3/2}}{3/2} = 26\,160 \text{ N}.$$

$$d_w = \bar{x} = \frac{\int \frac{x}{2} x dy}{\int x dy} = \frac{\frac{1}{2} \int_0^1 4x^3 dx}{\int_0^1 4x^2 dx} = \frac{1}{2} \left( \frac{1/4}{1/3} \right) = 0.375 \text{ m}.$$

$$\therefore P = \frac{1}{3} \times 39\,240 + \frac{0.375}{2} \times 26\,160 = 17\,980 \text{ N} \text{ or } \underline{17.98 \text{ kN}}.$$

b)  $H = 2.4 \text{ m}$ .  $F = 9810 \times 1.2 \times (2.4 \times 1.2) = 33903 \text{ N}$

$$\begin{aligned} W &= 9810 \int_0^{2.4} 1.2x dy = 9810 \times 1.2 \int_0^{2.4} \sqrt{\frac{y}{2}} dy \\ &= 8324 \times \frac{2}{3} \times (2.4)^{3/2} \\ &= 20633 \text{ N}. \end{aligned}$$



$$d_w = \bar{x} = \frac{\int \frac{x}{2} x dy}{\int x dy} = \frac{\frac{1}{2} \int_0^{2.4} \frac{y}{2} dy}{\int_0^{2.4} \sqrt{\frac{y}{2}} dy} = 0.411 \text{ m}$$

$$\therefore P = \frac{1}{2.4} \left( \frac{2.4}{3} 33903 + 0.411 \times 20633 \right) = 14834 \text{ N}$$

**Buoyancy**

2.78  $W$  = weight of displaced water.

a)  $20\,000 + 250\,000 = 9810 \times 3(6d + d^2/2)$ .  $\therefore d^2 + 12d - 18.35 = 0$ .  $\therefore d = \underline{1.372 \text{ m}}$ .

b)  $270\,000 = 1.03 \times 9810 \times 3(6d + d^2/2)$ .  $d^2 + 12d - 17.81 = 0$ .  $\therefore d = \underline{1.336 \text{ m}}$ .

2.79  $25 + F_B = 100$ .  $\therefore F_B = 75 = 9810 V$ .  $\therefore V = 7.645 \times 10^{-3} \text{ m}^3$

$$\gamma \times 7.645 \times 10^{-3} = 100. \quad \text{or } \underline{7645 \text{ cm}^3}$$

$$\therefore \gamma = \underline{13\,080 \text{ N/m}^3}.$$

2.80  $15000 \times 60 = 7.5 \times 90 \times \Delta d \times 9810$ .  $\Rightarrow \Delta d = 0.136 \text{ m} = 13.6 \text{ cm}$ .

2.81  $100\,000 \times 9.81 + 6\,000\,000 = (12 \times 30 + 8h \times 30) 9810$

$$\therefore h = 1.465 \text{ m}. \quad \therefore \text{distance from top} = 2 - 1.465 = \underline{0.535 \text{ m}}$$

2.82  $T + F_B = W$ . (See Fig. 2.11 c.)

$$T = 40\,000 - 1.59 \times 9810 \times 2 = 8804 \text{ N or } \underline{8.804 \text{ kN}}.$$

2.83 The forces acting on the balloon are its weight  $W$ , the buoyant force  $F_B$ , and the weight of the air in the balloon  $F_a$ . Sum forces:

$$F_B = W + F_a \quad \text{or} \quad \frac{4}{3} \pi R^3 \rho g = 1000 + \frac{4}{3} \pi R^3 \rho_a g$$

$$\frac{4}{3} \pi \times 5^3 \frac{100 \times 9.81}{0.287 \times 293} = 1000 + \frac{4}{3} \pi \times 5^3 \frac{100 \times 9.81}{0.287 T_a}. \quad \therefore T_a = 350.4 \text{ K or } \underline{77.4^\circ \text{C}}$$

2.84 The forces acting on the blimp are the payload  $F_p$ , the weight of the blimp  $W$ , the buoyant force  $F_B$ , and the weight of the helium  $F_h$ :

$$F_B = F_p + W + F_h$$

$$1500 \pi \times 150^2 \times \frac{100 \times 9.81}{0.287 \times 288} = F_p + 0.1 F_p + 1500 \pi \times 150^2 \times \frac{100 \times 9.81}{2.077 \times 288}$$

$$\therefore F_p = 9.86 \times 10^8 \quad \text{and} \quad N_{\text{people}} = \frac{9.86 \times 10^8}{800} = \underline{1.23 \times 10^6}$$

Of course equipment and other niceties such as gyms, pools, restaurants, etc., would add significant weight.

2.85 Neglect the bouyant force of air. A force balance yields

$$F_B = W + F$$

$$= 50 + 10 = 60 = 9800 \cancel{V}. \quad \therefore \cancel{V} = 0.006122 \text{ m}^3$$

Density:  $\rho g \cancel{V} = W.$

$$\rho \times 9.81 \times 0.006122 = 50. \quad \therefore \rho = \underline{832.5 \text{ kg/m}^3}$$

Specific wt:  $\gamma = \rho g = 832.5 \times 9.81 = \underline{8167 \text{ N/m}^3}$

Specific gravity:  $S = \frac{\rho}{\rho_{\text{water}}} = \frac{832.5}{1000} S = \underline{0.8325}$

2.86 From a force balance  $F_B = W + pA.$

a) The buoyant force is found as follows ( $h > 4.8 \text{ m}$ ):

$$\cos \theta = \frac{h - 4.5 - R}{R}, \text{ Area} = \theta R^2 - (h - 4.5 - R) R \sin \theta$$

$$\therefore F_B = 3 \times 9810 [\pi R^2 - \theta R^2 + (h - 4.5 - R) R \sin \theta].$$

$$F_B = 6670 + \gamma h A.$$

The  $h$  that makes the above 2  $F_B$ 's equal is found by trial-and-error:

$$h = 4.95: \quad 8263 \stackrel{?}{\approx} 7010 \quad h = 5.04: \quad 8295 \stackrel{?}{\approx} 8281$$

$$h = 5.1: \quad 8317 \stackrel{?}{\approx} 8685 \quad \therefore h = \underline{5.04 \text{ m}}.$$

b) Assume  $h > 4.9 \text{ m}$  and use the above equations with  $R = 0.4 \text{ m}$ :

$$h = 4.92: \quad 8249 \stackrel{?}{\approx} 8234 \quad \therefore h = \underline{4.92 \text{ m}}.$$

c) Assume  $h < 5 \text{ m}$ . With  $R = 0.5 \text{ m}$ ,

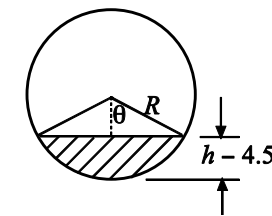
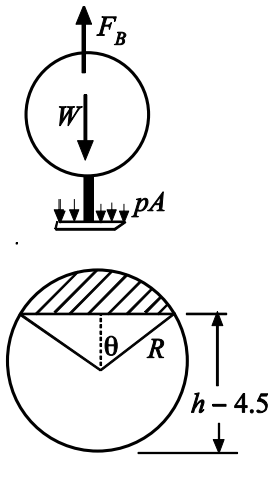
$$F_B = 3 \times 9810 [\theta R^2 - (R - h + 4.5) R \sin \theta]$$

$$F_B = 6670 + \gamma h A. \quad \cos \theta = \frac{R - h + 4.5}{R}$$

Trial-and-error for  $h$ :

$$h = 4.8: \quad 8221 \stackrel{?}{\approx} 6136 \quad h = 4.86: \quad 8239 \stackrel{?}{\approx} 7855$$

$$h = 4.92: \quad 8275 \stackrel{?}{\approx} 9572 \quad \therefore h = \underline{4.875 \text{ m}}.$$



2.87 a)  $W = F_B. \quad [0.01 + 13.6 \times 1000 \times h \pi \times 0.015^2 / 4] \times 9.81 = 9810 \cancel{V}.$

$$\cancel{V} = \frac{\pi \times 0.015^2}{4} \times 0.15 + \frac{\pi \times 0.005^2}{4} \times 0.06 = 2.769 \times 10^{-5} \text{ m}^3. \quad \therefore h = 7.361 \times 10^{-3} \text{ m}$$

$$\therefore m_{Hg} = 13.6 \times 1000 \times h\pi \times 0.015^2 / 4 = \underline{0.01769 \text{ kg}}$$

$$\text{b) } (0.01 + 0.01769) 9.81 = 9810 \left[ \frac{\pi \times 0.015^2}{4} \times 0.15 + \frac{\pi \times 0.005^2}{4} \times 0.12 \right] S_x. \quad \therefore S_x = \underline{0.959}$$

$$\text{c) } (0.01 + 0.01769) 9.81 = 9810 \frac{\pi \times 0.015^2}{4} \times 0.15 S_x. \quad \therefore S_x = \underline{1.045}$$

$$2.88 \quad (0.01 + m_{Hg}) 9.81 = 9810 \left[ \frac{\pi \times 0.015^2}{4} \times 0.15 + \frac{\pi \times 0.005^2}{4} \times 0.12 \right]. \quad \therefore m_{Hg} = 0.01886.$$

$$\text{a) } (0.01 + 0.01886) 9.81 = 9810 \frac{\pi \times 0.015^2}{4} \times 0.15 S_x. \quad \therefore S_x = \underline{1.089}.$$

$$\text{b) } m_{Hg} = \underline{0.01886 \text{ kg}}.$$

### Stability

$$2.89 \quad \text{a) } I_o = \frac{\pi d^4}{64} = \frac{\pi \times (0.25)^4}{64} = 1.92 \times 10^{-4} \text{ m}^4.$$

$$V = \frac{W}{\gamma_{H_2O}} = \frac{0.8 \times 9810 \times \pi \times (0.125)^2 \times 0.3}{9810} = 0.012 \text{ m}^3$$

$$\text{depth} = \frac{0.012}{\pi(0.125)^2} = 0.244 \text{ m}$$

$$\overline{GM} = \frac{1.92 \times 10^{-4}}{0.012} - (0.15 - 0.122) = -0.012 \text{ m}$$

$\therefore$  It will not float with ends horizontal.

$$\text{b) } I_o = 1.92 \times 10^{-4} \text{ m}^4$$

$$V = \frac{W}{\gamma_{H_2O}} = \frac{0.8 \times 9810 \times \pi \times (0.125)^2 \times 0.2}{9810} = 0.00785 \text{ m}^3$$

$$\text{depth} = \frac{0.00785}{\pi(0.125)^2} = 0.160 \text{ m}$$

$$\overline{GM} = \frac{1.92 \times 10^{-4}}{0.00785} - (0.10 - 0.080) = 0.0045 > 0$$

$\therefore$  In this case the cylinder will float with ends horizontal.

c)

$$V = \frac{W}{\gamma_{H_2O}} = \frac{0.8 \times 9810 \times \pi \times (0.125)^2 \times 0.25}{9810} = 0.00982 \text{ m}^3$$

$$\text{depth} = \frac{0.00982}{\pi(0.125)^2} = 0.2 \text{ m}$$

$$\overline{GM} = \frac{1.92 \times 10^{-4}}{0.00982} - (0.125 - 0.1) = -0.0054$$

∴ It will not float with ends horizontal.

2.90 With ends horizontal  $I_o = \pi d^4/64$ . The displaced volume is

$V = \gamma_x \pi d^2 h/4 \times 9800 = 8.014 \times 10^{-5} \gamma_x d^3$  since  $h = d$ . The depth the cylinder will sink is

$$\text{depth} = \frac{V}{A} = 8.014 \times 10^{-5} \gamma_x d^3 / \pi d^2 / 4 = 10.20 \times 10^{-5} \gamma_x d$$

The distance  $\overline{CG}$  is  $\overline{CG} = \frac{h}{2} - 10.2 \times 10^{-5} \gamma_x d / 2$ . Then

$$\overline{GM} = \frac{\pi d^4 / 64}{8.014 \times 10^{-5} \gamma_x d^3} - \frac{d}{2} + 10.2 \times 10^{-5} \gamma_x d / 2 > 0.$$

This gives (divide by  $d$  and multiply by  $\gamma_x$ ):  $612.5 - 0.5 \gamma_x + 5.1 \times 10^{-5} \gamma_x^2 > 0$ .

Consequently,  $\gamma_x > \underline{8369 \text{ N/m}^3}$  or  $\gamma_x < \underline{1435 \text{ N/m}^3}$

$$2.91 \quad V = \frac{W}{\gamma_{\text{water}}} = \frac{S \gamma_{\text{water}} d^3}{\gamma_{\text{water}}} = S d^3. \quad V = \frac{W}{\gamma_{\text{water}}} = \frac{S \gamma_{\text{water}} d^3}{\gamma_{\text{water}}} = S d^3. \quad \therefore h = Sd.$$

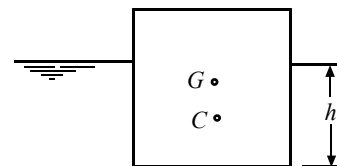
$$\overline{GM} = \frac{d^4/12}{Sd^3} - (d/2 - Sd/2) = d \left( \frac{1}{12S} - \frac{1}{2} + \frac{S}{2} \right).$$

If  $\overline{GM} = 0$  the cube is neutral and  $6S^2 - 6S + 1 = 0$ .

$$\therefore S = \frac{6 \pm \sqrt{36 - 24}}{12} = 0.7887, 0.2113.$$

The cube is unstable if  $\underline{0.2113 < S < 0.7887}$ .

Note: Try  $S = 0.8$  and  $S = 0.1$  to see if  $\overline{GM} > 0$ . This indicates stability.



2.92 As shown,  $\bar{y} = \frac{16 \times 9 + 16 \times 4}{16 + 16} = 6.5$  cm above the bottom edge.

$$G = \frac{4\gamma \times 9.5 + 16\gamma \times 8.5 + 16S_A \gamma \times 4}{0.5\gamma \times 8 + 2\gamma \times 8 + S_A \gamma \times 16} = 6.5 \text{ cm.}$$

$$\therefore 130 + 104 S_A = 174 + 64 S_A. \quad \therefore S_A = \underline{1.1}.$$

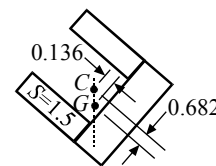
2.93 a)  $\bar{y} = \frac{16 \times 4 + 8 \times 1 + 8 \times 7}{16 + 8 + 8} = 4.$   $\bar{x} = \frac{16 \times 1 + 8 \times 4 + 8 \times 4}{16 + 8 + 8} = 2.5.$

$$\text{For } G: \quad y = \frac{1.2 \times 16 \times 4 + 0.5 \times 8 \times 1 + 1.5 \times 8 \times 7}{1.2 \times 16 + 0.5 \times 8 + 1.5 \times 8} = 4.682.$$

$$x = \frac{1.2 \times 16 + 0.5 \times 8 \times 4 + 1.5 \times 8 \times 4}{1.2 \times 16 + 0.5 \times 8 + 1.5 \times 8} = 2.364.$$

$G$  must be directly under  $C$ .

$$\tan \theta = \frac{0.136}{0.682}. \quad \therefore \theta = 11.3^\circ.$$



$$\begin{aligned} \text{b) } \bar{y} &= \frac{25 \times 5 + 12.5 \times 1.25 + 12.5 \times 8.75}{25 + 12.5 + 12.5} & \bar{x} &= \frac{25 \times 1.25 + 12.5 \times 5 + 12.5 \times 5}{25 + 12.5 + 12.5} \\ &= 5; & &= 3.125 \end{aligned}$$

$$\text{For } G: \quad y = \frac{1.2 \times 25 \times 5 + 0.5 \times 12.5 \times 1.25 + 1.5 \times 12.5 \times 8.75}{1.2 \times 25 + 0.5 \times 12.5 + 1.5 \times 12.5} = 5.85$$

$$x = \frac{1.2 \times 25 \times 1.25 + 0.5 \times 12.5 \times 5 + 1.5 \times 12.5 \times 5}{1.2 \times 25 + 0.5 \times 12.5 + 1.5 \times 12.5} = 2.95$$

$$\Delta y = 0.85, \quad \Delta x = 0.175. \quad \tan \theta = \frac{0.175}{0.85}. \quad \Rightarrow \quad \theta = 11.6^\circ.$$

2.94 The centroid  $C$  is 1.5 m below the water surface.  $\therefore \overline{CG} = 1.5$  m.

$$\text{Using Eq. 2.4.47: } \overline{GM} = \frac{\ell \times 8^3 / 12}{\ell \times 8 \times 3} - 1.5 = 1.777 - 1.5 = 0.277 > 0.$$

$\therefore$  The barge is stable.

$$2.95 \quad \bar{y} = \frac{8.485 \times 3.414 + 16.97 \times 1}{8.485 + 16.97} = 1.8 \text{ m.} \quad \therefore \overline{CG} = 1.8 - 1.5 = 0.3 \text{ m.}$$

$$\text{Using Eq. 2.4.47: } \overline{GM} = \frac{\ell \times 8.485^3 / 12}{34.97 \ell} - 0.3 = 1.46 - 0.3 = 1.16. \quad \therefore \text{Stable.}$$

### Linearly Accelerating Containers

$$2.96 \quad \text{a) } \tan \alpha = \frac{20}{9.81} = \frac{H}{4}. \quad \therefore H = 8.155 \text{ m.} \quad p_{\max} = 9810 (8.155 + 2) = 99\,620 \text{ Pa} = 99.6 \text{ kPa}$$

$$\text{b) } p_{\max} = \rho(g + a_z) h = 1000 (9.81 + 20) \times 2 = 59\,620 \text{ Pa} = 59.6 \text{ kPa}$$

$$\text{c) } p_{\max} = 1000 \times 18 (3.6) + 1000 (9.81 + 18) \times (1.8) = 114\,860 \text{ Pa} = 115 \text{ kPa}$$

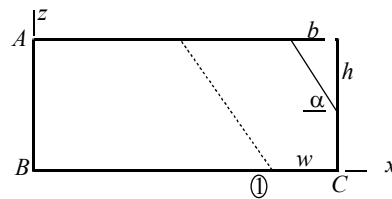
$$\text{d) } p_{\max} = 1000 \times (9.81 + 18) \times 1.8 = 50\,060 \text{ Pa} = 50.1 \text{ kPa}$$

2.97 The air volume is the same before and after.

$$\therefore 0.5 \times 8 = hb/2. \quad \tan \alpha = \frac{10}{9.81} = \frac{h}{b}$$

$$4 = \frac{h}{2} \frac{9.81}{10} h. \quad \therefore h = 2.856. \quad \therefore \text{Use dotted line.}$$

$$2.5w + \frac{1}{2} \times 2.5 \times 2.452 = 4. \quad \therefore w = 0.374 \text{ m.}$$



a)  $p_A = -1000 \times 10 (0 - 7.626) - 1000 \times 9.81 \times 2.5 = 51\,740 \text{ Pa}$  or 51.74 kPa

b)  $p_B = -1000 \times 10 (0 - 7.626) = 76\,260 \text{ Pa}$  or 76.26 kPa

c)  $p_C = 0$ . Air fills the space to the dotted line.

2.98 Use Eq. 2.5.2: Assume an air-water surface as shown in the above figure.

a)  $60\,000 = -1000 a_x (0-8) - 1000 \times 9.81 \left[ 0 - \left( 2.5 - \sqrt{\frac{8a_x}{9.81}} \right) \right]$

$$4 = \frac{h^2 \times 9.81}{2a_x} \quad 60 = 8 a_x + 24.52 - 9.81 \sqrt{\frac{8a_x}{9.81}} \quad \text{or} \quad a_x - 4.435 = 1.1074 \sqrt{a_x}$$

$$a_x^2 - 10.1 a_x + 19.67 = 0 \quad \therefore a_x = 2.64, \underline{7.46 \text{ m/s}^2}$$

b)  $60\,000 = -1000 a_x (-8) - 1000 (9.81 + 10) \left( -2.5 + \sqrt{\frac{8a_x}{9.81}} \right)$

$$60 = 8 a_x + 49.52 - 19.81 \sqrt{\frac{8a_x}{19.81}} \quad \text{or} \quad a_x - 1.31 = 1.574 \sqrt{a_x}$$

$$a_x^2 - 5.1 a_x + 1.44 = 0 \quad \therefore a_x = 0.25, \underline{4.8 \text{ m/s}^2}$$

c)  $60\,000 = -1000 a_x (-8) - 1000 (9.81 + 5) \left( -2.5 + \sqrt{\frac{8a_x}{14.81}} \right)$

$$60 = 8 a_x + 37.0 - 14.81 \sqrt{\frac{8a_x}{14.81}} \quad \text{or} \quad a_x - 2.875 = 1.361 \sqrt{a_x}$$

$$a_x^2 - 7.6 a_x + 8.266 = 0 \quad \therefore a_x = 1.32, \underline{6.28 \text{ m/s}^2}$$

2.99 a)  $a_x = 20 \times 0.866 = 17.32 \text{ m/s}^2$ ,  $a_z = 10 \text{ m/s}^2$ . Use Eq. 2.5.2 with the peep hole as position 1. The  $x$ -axis is horizontal passing thru  $A$ . We have

$$p_A = -1000 \times 17.32 (0 - 1.232) - 1000 (9.81 + 10) (0 - 1.866) = \underline{58\,290 \text{ Pa}}$$

b)  $p_A = -1000 \times 8.66 (0 - 1.848) - 1000 (9.81 + 5) (0 - 2.799) = \underline{57\,460 \text{ Pa}}$

2.100 a) The pressure on the end  $AB$  ( $z$  is zero at  $B$ ) is, using Eq. 2.5.2,

$$p(z) = -1000 \times 10 (-7.626) - 1000 \times 9.81(z) = 76\,260 - 9810 z$$

$$\therefore F_{AB} = \int_0^{2.5} (76\,260 - 9810z) 4 dz = 640\,000 \text{ N} \quad \text{or} \quad \underline{640 \text{ kN}}$$

b) The pressure on the bottom  $BC$  is



$$p(x) = -1000 \times 10 (x - 7.626) = 76\,260 - 10\,000x.$$

$$\therefore F_{BC} = \int_0^{7.626} (76\,260 - 10\,000x)4dx = 1.163 \times 10^6 \text{ N or } \underline{1163 \text{ kN}}$$

c) On the top  $p(x) = -1000 \times 10 (x - 5.174)$  where position 1 is on the top surface:

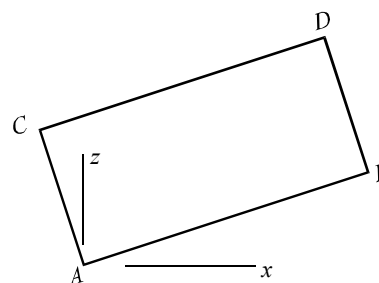
$$\therefore F_{top} = \int_0^{5.174} (51\,740 - 10\,000x)4dx = 5.35 \times 10^5 \text{ N or } \underline{535 \text{ kN}}$$

2.101 a) The pressure at  $A$  is 58.29 kPa. At  $B$  it is

$$p_B = -1000 \times 17.32 (1.732 - 1.232) - 1000 (19.81) (1 - 1.866) = 8495 \text{ Pa.}$$

Since the pressure varies linearly over  $AB$ , we can use an average pressure times the area:

$$F_{AB} = \frac{58\,290 + 8495}{2} \times 1.5 \times 2 = 100\,200 \text{ N or } \underline{100.2 \text{ kN}}$$



b)  $p_D = 0$ .  $p_C = -1000 \times 17.32 (-0.5 - 1.232) - 1000 \times 19.81 (0.866 - 1.866) = 49\,810 \text{ Pa.}$

$$F_{CD} = \frac{1}{2} \times 49\,810 \times 1.5 \times 2 = 74\,720 \text{ N or } \underline{74.72 \text{ kN.}}$$

c)  $p_A = 58\,290 \text{ Pa.}$   $p_C = 49\,810 \text{ Pa.}$   $\therefore F_{AC} = \frac{58.29 + 49.81}{2} \times 1.5 = \underline{81.08 \text{ kN.}}$

2.102 Use Eq. 2.5.2 with position 1 at the open end:

a)  $p_A = 0$  since  $z_2 = z_1$ .

$$p_B = 1000 \times 19.81 \times 0.6 = 11\,890 \text{ Pa} = \underline{11.9 \text{ kPa}}$$

$$p_C = 11\,890 \text{ Pa} = \underline{11.9 \text{ kPa}}$$

b)  $p_A = -1000 \times a_x = 10 \text{ m/s}^2 (0.9 - 0) = -9000 \text{ Pa} = \underline{-9 \text{ kPa}}$

$$p_B = -1000 \times 10 (0.9) - 1000 \times 9.81 (-0.6) = -3114 \text{ Pa} = \underline{-3.11 \text{ kPa}}$$

$$p_C = -1000 \times 9.81 \times (-0.6) = 5886 \text{ Pa} = \underline{5.89 \text{ kPa}}$$

c)  $p_A = -1000 \times 20 (0.9) = -18\,000 \text{ Pa} = \underline{-18 \text{ kPa}}$

$$p_B = -1000 \times 20 \times 0.9 - 1000 \times 19.81 (-0.6) = -6110 \text{ Pa} = \underline{-6.11 \text{ kPa}}$$
  $p_C = 11.9 \text{ kPa}$

d)  $p_A = 0$ .  $p_B = 1000 \times (9.81 - 18) \times 0.625 = -5119 \text{ Pa} = \underline{-5.12 \text{ kPa.}}$   $p_C = -5119 \text{ Pa} = \underline{-5.12 \text{ kPa}}$

e)  $p_A = 1000 \times 18 \times (-1.5 \times 0.625) = -16.87 \text{ kPa}$

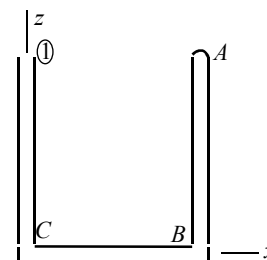
$$p_B = 1000 \times 18 \times (-1.5 \times 0.625) - 1000 \times 9.81 \times (-0.625) = \underline{-10.74 \text{ kPa}}$$

$$p_C = -1000 \times 9.81 \times (-0.625) = \underline{6.13 \text{ kPa}}$$

f)  $p_A = 1000 \times (-9) \times (-1.5 \times 0.625) = \underline{8.44 \text{ kPa}}$

$$p_B = 1000 \times (-9) \times (1.5 \times 0.625) - 1000 \times 18.81 \times (-0.625) = \underline{3.32 \text{ kPa}}$$

$$p_C = -1000 \times 18.81 \times (-0.625) = \underline{11.76 \text{ kPa}}$$



**Rotating Containers**

2.103 Use Eq. 2.6.4 with position 1 at the open end:

$$\omega = \frac{50 \times 2\pi}{60} = 5.236 \text{ rad/s.}$$

a)  $p_A = (1000 \times 5.236^2 / 2) \times (0.6 \times 1.5)^2 = \underline{11.1 \text{ kPa}}$

$$p_B = \frac{1}{2} \times 1000 \times 5.236^2 \times 0.9^2 + 9810 \times 0.6 = \underline{17.0 \text{ kPa}}$$

$$p_C = 9810 \times 0.6 = \underline{5.89 \text{ kPa}}$$

b)  $p_A = \frac{1}{2} \times 1000 \times 5.236^2 \times 0.6^2 = \underline{4.93 \text{ kPa}}$

$$p_B = \frac{1}{2} \times 1000 \times 5.236^2 \times 0.6^2 + 9810 \times 0.4 = \underline{8.86 \text{ kPa}}$$

$$p_C = 9810 \times 0.4 = \underline{3.92 \text{ kPa}}$$

c)  $p_A = \frac{1000 \times 5.236^2}{2} \times 0.45^2 = \underline{2.78 \text{ kPa}}$

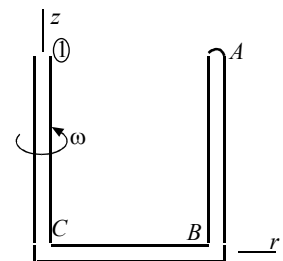
$$p_B = \frac{1000 \times 5.236^2}{2} \times 0.45^2 + 9810 \times 0.3 = \underline{5.72 \text{ kPa}}$$

$$p_C = 9810 \times 0.3 = \underline{2.94 \text{ kPa}}$$

d)  $p_A = \frac{1000 \times 5.236^2}{2} \times 0.375^2 = \underline{1.93 \text{ kPa}}$

$$p_B = \frac{1000 \times 5.236^2}{2} \times 0.375^2 + 9810 \times 0.25 = \underline{4.38 \text{ kPa}}$$

$$p_C = 9810 \times 0.25 = \underline{2.45 \text{ kPa}}$$



2.104 Use Eq. 2.6.4 with position 1 at the open end.

a)  $p_A = \frac{1}{2} \times 1000 \times 10^2 (0 - 0.9^2) = \underline{-40.5 \text{ kPa.}}$

$$p_B = -40\,500 + 9810 \times 0.6 = \underline{-34.6 \text{ kPa.}}$$

$$p_C = 9810 \times 0.6 = \underline{5.89 \text{ kPa.}}$$

b)  $p_A = \frac{1}{2} \times 1000 \times 10^2 (0 - 0.6^2) = \underline{-18.0 \text{ kPa.}}$

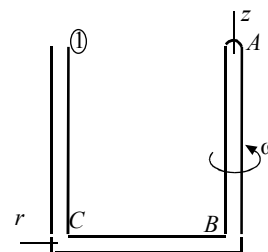
$$p_B = -18\,000 + 9810 \times 0.4 = \underline{-14.1 \text{ kPa.}}$$

$$p_C = 9810 \times 0.4 = \underline{3.92 \text{ kPa.}}$$

c)  $p_A = \frac{1000 \times 10^2}{2} (0 - 0.45^2) = \underline{-10.1 \text{ kPa}}$

$$p_B = -10\,100 \text{ Pa} + 9810 \times 0.3 = \underline{-7.18 \text{ kPa}}$$

$$p_C = 9810 \times 0.3 = \underline{2.94 \text{ kPa}}$$



$$d) p_A = \frac{1000 \times 10^2}{2} (0 - 0.375^2) = \boxed{-7.03 \text{ kPa}}$$

$$p_B = -10 \ 100 \text{ Pa} + 9810 \times 0.25 = \boxed{-4.58 \text{ kPa}}$$

$$p_B = 9810 \times 0.25 = \boxed{2.45 \text{ kPa}}$$

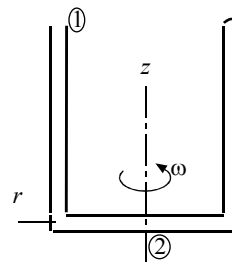
2.105 Use Eq. 2.6.4 with position 1 at the open end and position 2 at the origin. Given:  $p_2 = 0$ .

$$a) 0 = \frac{1}{2} \times 1000 \omega^2 (0 - 0.45^2) - 9810 (0 - 0.6). \quad \therefore \omega = \underline{7.62 \text{ rad/s.}}$$

$$b) 0 = \frac{1}{2} \times 1000 \omega^2 (0 - 0.3^2) - 9810 (0 - 0.4). \quad \therefore \omega = \underline{9.34 \text{ rad/s.}}$$

$$c) 0 = \frac{1}{2} \times 1000 \omega^2 (0 - 0.225^2) - 9810 (0 - 0.3). \quad \therefore \omega = \underline{10.8 \text{ rad/s.}}$$

$$d) 0 = \frac{1}{2} \times 1000 \omega^2 (0 - 0.1875^2) - 9810 (0 - 0.25). \quad \therefore \omega = \underline{11.8 \text{ rad/s.}}$$



2.106 The air volume before and after is equal.

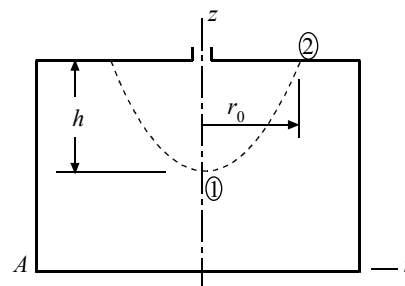
$$\therefore \frac{1}{2} \pi r_0^2 h = \pi \times 6^2 \times 2. \quad \therefore r_0^2 h = 0.144.$$

$$a) \text{ Using Eq. 2.6.5: } r_0^2 \times 5^2 / 2 = 9.81 h$$

$$\therefore h = 0.428 \text{ m}$$

$$\therefore p_A = \frac{1}{2} \times 1000 \times 5^2 \times 0.6^2 - 9810 (-0.372)$$

$$= \underline{8149 \text{ Pa.}}$$



$$b) r_0^2 \times 7^2 / 2 = 9.81 h. \quad \therefore h = 0.6 \text{ m.}$$

$$\therefore p_A = \frac{1000}{2} \times 7^2 \times 0.6^2 + 9810 \times 0.2 = \underline{10 \ 780 \text{ Pa.}}$$

c) For  $\omega = 10$ , part of the bottom is bared.

$$\pi \times 6^2 \times 2 = \frac{1}{2} \pi r_0^2 h - \frac{1}{2} \pi r_1^2 h_1.$$

Using Eq. 2.6.5:

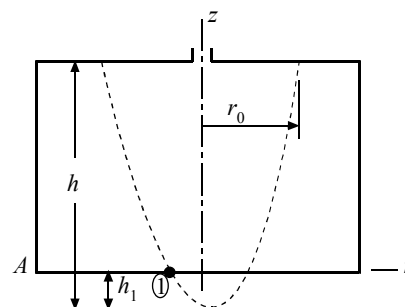
$$\frac{\omega^2 r_0^2}{2g} = h, \quad \frac{\omega^2 r_1^2}{2g} = h_1.$$

$$\therefore 0.144 = \frac{2g}{\omega^2} h^2 - \frac{2g}{\omega^2} h_1^2 \quad \text{or}$$

$$h^2 - h_1^2 = \frac{0.144 \times 10^2}{2 \times 9.81}.$$

$$\text{Also, } h - h_1 = 0.8. \quad 1.6h - 0.64 = 0.7339. \quad \therefore h = 0.859 \text{ m, } r_1 = 0.108 \text{ m.}$$

$$\therefore p_A = 1000 \times 10^2 (0.6^2 - 0.108^2) / 2 = \underline{17 \ 400 \text{ Pa.}}$$



d) Following part (c):  $h^2 - h_1^2 = \frac{0.144 \times 20^2}{2 \times 9.81}$ .  $1.6h - 0.64 = 2.936$ .  $\therefore h = 2.235$  m.

$\therefore p_A = 1000 \times 20^2 (0.6^2 - 0.265^2)/2 = \underline{57\,900 \text{ Pa}}$   $r_1 = 0.265$  m

2.107 The answers to Problem 2.106 are increased by 25 000 Pa.

a) 33 150 Pa      b) 35 780 Pa      c) 42 400 Pa      d) 82 900 Pa

2.108  $p(r) = \frac{1}{2} \rho \omega^2 r^2 - \rho g [0 - (0.8 - h)]$ .

$p(r) = 500 \omega^2 r^2 + 9810(0.8 - h)$  if  $h < 0.8$ .

$p(r) = 500 \omega^2 (r^2 - r_1^2)$  if  $h > 0.8$ .

a)  $F = \int p 2\pi r dr = 2\pi \int_0^{0.6} (12\,500r^3 + 3650r) dr = \underline{6670 \text{ N}}$ .

(We used  $h = 0.428$  m)

b)  $F = \int p 2\pi r dr = 2\pi \int_0^{0.6} (24\,500r^3 + 1962r) dr = \underline{7210 \text{ N}}$ . (We used  $h = 0.6$  m)

c)  $F = \int p 2\pi r dr = 2\pi \int_{-0.108}^{0.6} (50\,000(r^3 - 0.108^2 r)) dr = \underline{9530 \text{ N}}$ . (We used  $r_1 = 0.108$  m)

d)  $F = \int p 2\pi r dr = 2\pi \int_{0.265}^{0.6} (200\,000(r^3 - 0.265^2 r)) dr = \underline{26\,400 \text{ N}}$ . (We used  $r_1 = 0.265$  m)

